NORMAL SHOCK – NUMERICALS ①

Points to Remember

1) $h_{ox} = h_{oy} \Rightarrow T_{ox} = T_{oy}$

$x \rightarrow$ upstream of shock

$y \rightarrow$ downstream of shock

2) $P_{ox} \neq P_{oy}$

3) $\dot{m} = \rho_x A C_x = \rho_y A C_y$

4) Knowing M_x or M_y (anyone) is important, M_y depends on M_x

5) $P = \rho R T$, $M_x = \frac{C_x}{\sqrt{\gamma R T_x}}$, $M_y = \frac{C_y}{\sqrt{\gamma R T_y}}$

6) If nothing is specified, consider flow before and after shock has isentropic.

7) To find stagnation condition (P_o, T_o) at any point in the flow use isentropic tables.

8) Gas table and scientific calculator's play very important role.

Q). A gas ($\gamma = 1.4$, $R = 0.287$ kJ/kgK) at Mach number of 1.8, $P = 0.8$ bar & $T = 373$K passes through a normal shock. Determine density after shock. Compare this value in a isentropic compression through same pressure ratio.

Sol: Given

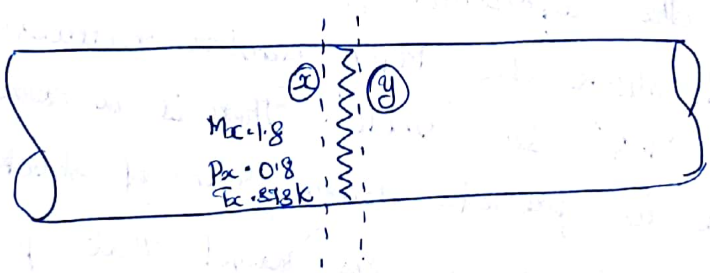
$\gamma = 1.4$

$R = 287$ J/kgK

$M_x = 1.8$

$P_x = 0.8$ bar

$T_x = 373$K



For $\gamma = 1.4$, $M_x = 1.8$ from Normal shock tables (Page no: __)

M_x	M_y	P_y/P_x	T_y/T_x	P_{oy}/P_{ox}	P_{oy}/P_x
1.8	0.616	3.613	1.532	0.813	4.670

Therefore – $M_y = 0.616$

$$\frac{P_y}{P_x} = 3.613 \Rightarrow P_y = 3.613 \times P_x$$

$$P_y = 3.613 \times 0.8 = 2.890 \text{ bar}$$

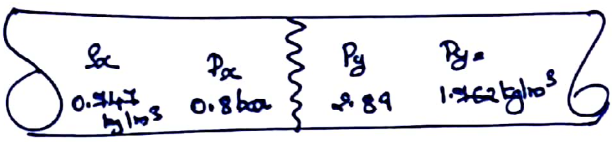
$$\frac{T_y}{T_x} = 1.582 \Rightarrow T_y = 1.582 \times T_x$$

$$T_y = 1.582 \times 373 = 591.44\,K$$

Applying $P = \rho R T$ ~~at~~ down stream of shock. we get

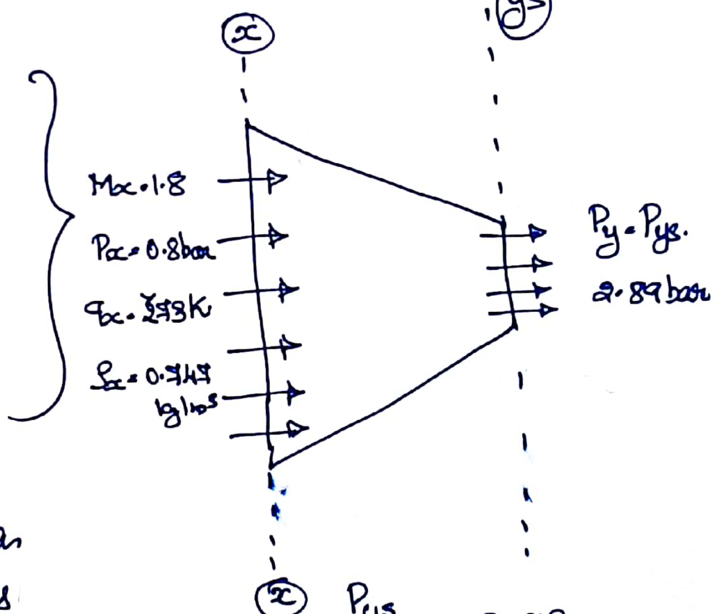
$$P_y = \rho_y R T_y$$

$$\rho_y = \frac{P_y}{R T_y} = \frac{[2.890 \text{ bar} \times 10^5]\,N/m^2}{287 \times 591.44} = \boxed{1.762\,kg/m^3}$$



Pressure ratio Across normal shock

$\left\{ \rho_x = \frac{P_x}{R T_x} = 0.747\,kg/m^3 \right\}$ $\quad \frac{P_y}{P_x} = 3.613$



$$\frac{P_{ys}}{P_x} = 3.613$$

When the supersonic flow encounters an normal shock the Mach number reduces from '1.8' to '0.616'. There is a corresponding increase in pressure down stream of shock (shock is a compression wave). From calculation it was found that pressure increases from "$P_x = 0.8$ bar" to "$P_y = 2.89$ bar" when the fluid passes through normal shock. Hence pressure ratio $\frac{P_y}{P_x} = 3.613$.

Consider the same flow upstream of shock., but instead of a constant area duct with normal shock, the flow passes through a supersonic diffusor as a result of which pressure increases by isentropic compression so after passing through this diffuser the pressure of fluid at exit becomes 2.89 bar i.e.. same as that obtained in downstream of shock, but through isentropic flow through a diffuser, then pressure ratio in diffuser i.e.. $\frac{P_{ys}}{P_x} = 3.613$ $\left\{ \frac{\text{exit pressure}}{\text{inlet pressure}} \text{ - Press ratio} \right\}$

To calculate density at the exit due to isentropic compression we should know Mach number at the exit of this isentropic flow.

From inlet condition we have.

$M_x = 1.8$, $P_x = 0.8$ bar, $T_x = 378K$.

For $\gamma = 1.4$, $R = 287$ J/kgK, $M = 1.8$ from isentropic tables

M_x	M_x^*	$\frac{T_x}{T_{ox}}$	$\frac{P_x}{P_{ox}}$	$\frac{A_x}{A^*}$
1.8	1.586	0.607	0.174	1.439

$$\frac{T_x}{T_{ox}} = 0.609 \Rightarrow T_{ox} = \frac{T_x}{0.609} = \frac{378}{0.607} = 614.497\,K$$

$$\frac{P_x}{P_{ox}} = 0.174 \Rightarrow P_{ox} = \frac{P_x}{0.174} = \frac{0.8}{0.174} = 4.598 \text{ bar.}$$

$$\left.\begin{aligned} P_{ox} &= P_{oys} \\ T_{ox} &= T_{oy} \end{aligned}\right\} \text{ isentropic compression through diffuser.}$$

We know at the exit $P_{ys} = 2.89$ bar & $P_{oys} = 4.598$ bar

Therefore at the exit $\left(\frac{P}{P_o}\right)_{ys} = \frac{2.89}{4.598} = 0.628$

$\gamma = 1.4$, $P/P_o = 0.628$ from isentropic tables we get

M_{ys}	$\frac{T_{ys}}{T_{oys}}$	$\frac{P_{ys}}{P_{oys}}$
0.84	0.876	0.680

4

$$\frac{T_{ys}}{T_{0ys}} = 0.876 \Rightarrow T_{ys} = 0.876 \times T_{0ys}$$

$$= 0.876 \times 614.497 = 538.299$$

$$\frac{P_{ys}}{P_{0ys}} = 0.620 \Rightarrow P_{ys} = 0.630 \times 4.598 = 2.896 \text{ bar}$$

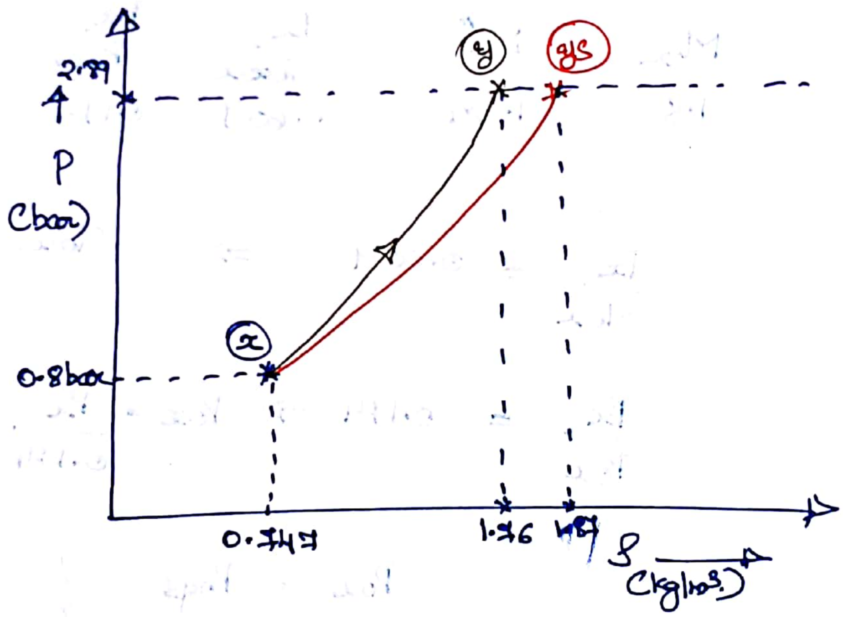
(already known)

Applying $p = \rho RT$ at exit of isentropic compression flow.

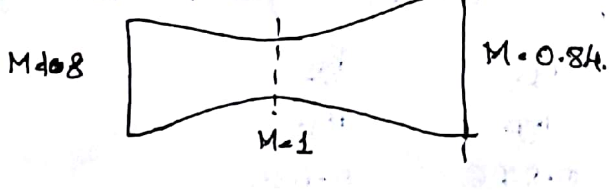
$$P_{ys} = \rho_{ys} \times R \times T_{ys}$$

$$\rho_{ys} = \frac{P_{ys}}{R \times T_{ys}} = \frac{(2.89 \times 10^5)\, N/m^2}{287 \times 588.299} = \boxed{1.871 \text{ kg/m}^3}$$

We see that for same given condition under the same pressure ratio isentropic compression value is higher than that obtained through normal shock. This is because normal shock reduces available energy & there is increase in entropy i.e. loss, where has isentropic compression there is no entropy change. hence losses are zero.



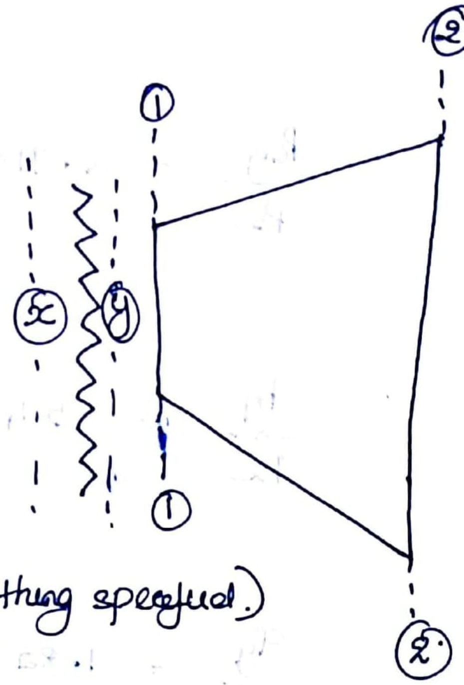
Note: We see that through isentropic compression for given pressure ratio exit Mach number is 0.84 which is subsonic. Hence supersonic diffuser in this case should be a C-D duct to decelerate flow from super sonic to subsonic



* The ratio of exit to entry area in a subsonic diffuser is 4.0. The Mach number of jet of air approaching diffuser at $P_0 = 1.018$ bar, $T = 290K$ is 2.2. There is a standing normal shock wave just outside the diffuser entry. The flow in diffuser is isentropic. Determine at the exit of diffuser a) Mach number, b) Temperature, c) Pressure. What is the stagnation pressure loss between initial and final states of flow?

Sol: Given

Area ratio of diffuser $\frac{A_2}{A_1} = 4$



Mach no. upstream $M_x = 2.2$

Stagnation pressure upstream $\}$ $P_{0x} = 1.018$ bar

Static temperature upstream $\}$ $T_x = 290K$

$\gamma = 1.4$, $R = 287$ J/kgK (nothing specified)

Conditions upstream of shock is (x).

Conditions downstream of shock is (y)

Conditions at inlet of diffuser is (1)

Conditions at outlet of diffuser is (2)

Since nothing is specified we can consider isentropic flows before & after the shock. Flow through diffuser is isentropic (given).

Therefore conditions at (y) will be equal to conditions at (1)

To know the conditions at the exit of diffuser we need inlet conditions which is same that ~~up~~ downstream conditions of normal shock.

Hence we need to start with normal shock obtain ~~upst~~ downstream values and use it to find exit condition assuming isentropic flow.

Normal shock related Calculations

For $\gamma = 1.4$ $M_x = 2.2$ from Normal shock tables

M_x	M_y	P_y/P_x	T_y/T_x	$\frac{P_{oy}}{P_{ox}}$	$\frac{P_{oy}}{P_x}$
2.2	0.547	5.48	1.857	0.628	6.716

$$M_y = 0.547$$

$$\frac{P_{oy}}{P_{ox}} = 0.628 \Rightarrow P_{oy} = 0.628 \times P_{ox}$$

$$P_{oy} = 0.628 \times 1.018 = 0.686 \text{ bar}$$

$$\frac{P_{oy}}{P_x} = 6.716 \Rightarrow P_x = P_{oy}/6.716$$

$$P_x = 0.686/6.716 = 0.0947 \text{ bar}$$

$$\frac{P_y}{P_x} = 5.480 \Rightarrow P_y = 5.480 \times P_x$$

$$P_y = 5.480 \times 0.0947 = 0.519 \text{ bar}$$

$$\frac{T_y}{T_x} = 1.857 \Rightarrow T_y = 1.857 \times T_x$$

$$T_y = 1.857 \times 290 = 538.53 \text{ K}$$

Hence we have.

$$M_1 = M_y = 0.547$$

$$T_1 = T_y = 538.53 \text{ K}$$

$$P_1 = P_y = 0.519 \text{ bar}$$

$$P_{01} = P_{oy} = 0.686 \text{ bar}$$

For $\gamma = 1.4$, $M = 2.2$ from isentropic tables $\frac{T_x}{T_{ox}} = 0.508$

$$T_x/T_{ox} = 0.508 \Rightarrow T_{ox} = T_x/0.508 \quad 570.866\text{K}$$

$$= 290/0.508 = \cancel{549.24\text{K}}$$

$$T_{ox} = T_{oy} = T_{01} = \cancel{549.24\text{K}}\ 570.866\text{K}.$$

7

calculate values at ② we need to find out Mach number at ②. The data we have for this purpose is $\frac{A_2}{A_1} = 4$

$$\frac{A_2}{A_1} = \frac{A_2/A^*}{A_1/A^*}$$

$$\frac{A_2}{A^*} = \frac{A_2}{A_1} \times \frac{A_1}{A^*}$$

To calculate A_1/A^* we need values from isentropic tables corresponding to Mach number $M_1 = 0.547$.

For $\gamma = 1.4$, $M_1 = 0.55$ from isentropic tables.

M_1	T_1/T_{01}	P_1/P_{01}	A_1/A^*
0.55	0.943	0.814	1.255

Therefore $\frac{A_2}{A^*} = \frac{A_2}{A_1} \times \frac{A_1}{A^*} = 4 \times 1.255 = 5.02$

For $\gamma = 1.4$, $A/A^* = 5.02$ from isentropic tables we get two Mach number (i) $\approx 0.11 - 0.12$ (ii) $\approx 3.15 - 3.2$.

One is supersonic and the other is subsonic. For subsonic diffuser inlet & exit Mach numbers remains subsonic. Therefore we take (i)

M	T/T_0	P/P_0	A/A^*
0.11	0.9976	0.992	5.299
0.116	0.9974	0.990	5.02
0.12	0.9971	0.989	4.864

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

$$\frac{M - 0.11}{0.12 - 0.11} = \frac{\overset{(5.02)}{A/A^*} - 5.299}{\underbrace{4.864 - 5.299}_{0.6414}}$$

$\Rightarrow M = 0.1164$

$M_2 = \boxed{0.116}$

$\frac{T_2}{T_{02}} = 0.9974 \Rightarrow T_2 = 0.9974 \times T_{02}$

$T_2 = 0.9974 \times 570.866 = \boxed{569.394\ K}$

8

$$\frac{P_2}{P_{02}} = 0.990 \Rightarrow P_2 = 0.990 \times P_{02}$$

$$P_2 = 0.990 \times 0.636 = \boxed{0.6296 \text{ bar}}$$

$$P_{01} = P_{02} = 0.686 \text{ bar} \quad \{\text{Isentropic flow}\}$$

stagnation pressure loss from initial & final state of flow:

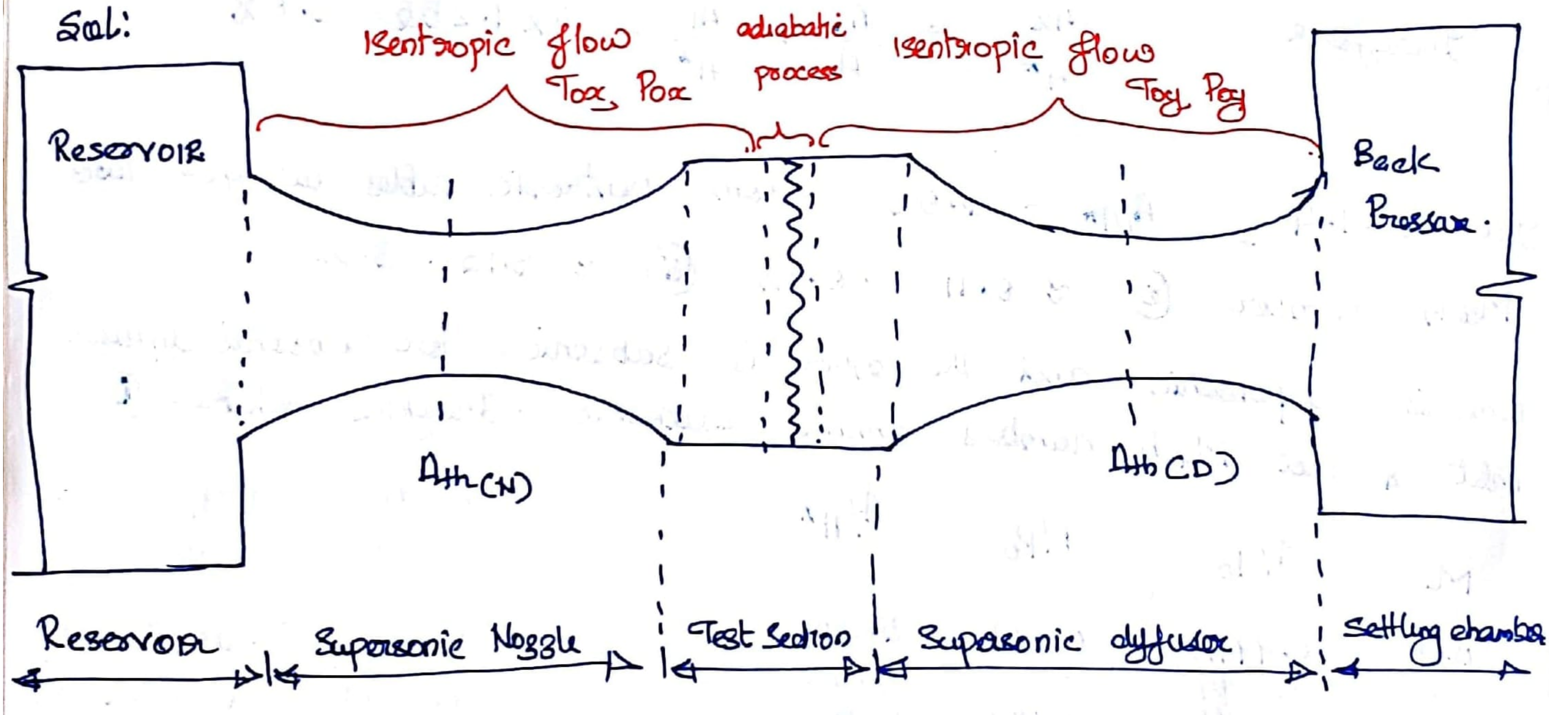
$$\Delta P_0 = P_{0x} - P_{02} = 1.018 - 0.686 = 0.8884 \text{ bar}$$

$$\boxed{= 0.88 \text{ bar}}$$

Q8) Following data refers to a supersonic wind tunnel:
- nozzle throat area = 200 cm²
- Test section cross section = 337.5 cm²
- Working fluid = air ($\gamma = 1.4$, $R = 287$ J/kgK)

Determine Test section Mach number & diffuser throat area if a normal shock is located at test section.

Sol:



A supersonic wind tunnel aims at generating supersonic flow in the test section. For this air from reservoir is accelerated using supersonic nozzle. Test are conducted with supersonic flow at test section. After this low pressure high velocity fluid is compressed through a supersonic diffuser and raised to ambient & expelled out. To maintain this flow the back pressure is lowered using vacuum pump.

9

Sol: supersonic flow is being set occurance of shock is common case th dropping of back pressure to suitable operating values we get shock free supersonic flow. At this point we can say that the entire flow is isentropic. But if shock appears in the flow it divides the entire flow into two isentropic flows - ① upstream of shock ② downstream of shock.

In this problem a normal shock occurs in the test ~~test~~ section., for the upstream stream flow A^* occurs at nozzle throat and downstream flow A^* occurs at diffuser throat.

$$\text{here} \quad T_{0x} = T_{0y}$$

$$P_{0x} \neq P_{0y}$$

Shock can occur only in supersonic flow. If shock occurs in test section it means that flow is supersonic in test section. If flow is supersonic in test section, it should be supersonic at nozzle exit (same area, no change). For C-D nozzle to have supersonic flow at exit Mach number at throat should be unity.

$$\text{Therefore} \quad A_{th(CN)} = A^*_x$$

From the question we have. $\dfrac{A_{test}}{A_{th}} = \left(\dfrac{A}{A^*}\right)_x = \dfrac{337.5}{200} = 1.6875$

For $\gamma = 1.4$, $\dfrac{A}{A^*} = 1.6875$ from isentropic tables $M \approx 0.375$, $M \approx 2.000$

M_x	T_x/T_{0x}	P_y/P_{0x}	A_x/A_x^*
2	0.555	0.128	1.687

Taking supersonic value.

Test section Mach number before shock = $\boxed{2.00}$

To get upstream data we need to use shock tables.

For $\gamma = 1.4$ $M_x = 2$ from normal shock tables

M_x	M_y	P_y/P_x	T_y/T_x	P_{oy}/P_{ox}
2	0.577	4.5	1.687	0.721

Mach number in test section after shock = $\boxed{0.577}$

In the upstream the only point where $M=1$ occurs is diffuser throat (minimum cross section point)

Therefore we take $A_y^* = A_{th(D)}$

Test section area will be the area for diffuser inlet.

$$\frac{A_{test}}{A_{th(D)}} = \left(\frac{A}{A^*}\right)_y \Rightarrow \text{Can be obtained from isentropic tables.}$$

For $\gamma = 1.4$, $M = 0.577 \simeq 0.58$ from isentropic tables

M_y	A_y/A_y^*
0.58	1.213

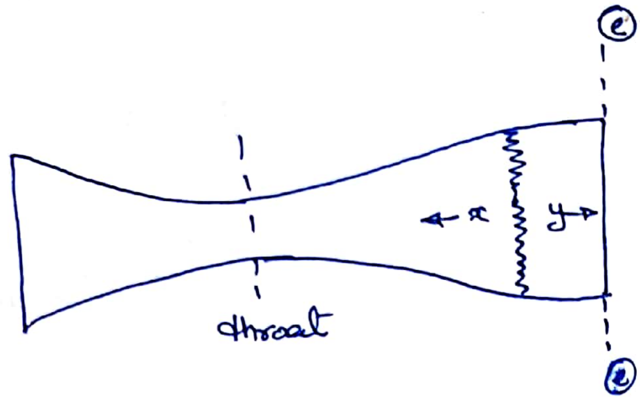
$$\frac{A_y}{A_y^*} = 1.213 \Rightarrow A_y^* = \frac{A_y}{1.213}$$

$$= \frac{337.5}{1.213} = 278.236\ cm^2$$

$\Rightarrow$ $A_{th(D)} = A_y^* = \boxed{278.24\ cm^2}$

Normal shock occurs in the diverging section of a C-D nozzle. (11)
The throat area is 1/3 of exit area and static pressure at exit is 0.4 times stagnation pressure at entry.
Determine a) Mach numbers b) static pressure c) C.S.A at shock occurs.

Sol:



For shock to occur in diverging section flow should have been supersonic. This means at throat Mach number is unity and nozzle is choked.

$\therefore A_{th} = A^*$

Taking $\gamma = 1.4$, $R = 287 \ J/kg K$

$$\frac{\dot{m}}{A^*}\frac{\sqrt{T_0}}{P_0} = 0.0404$$

given

$$A_{th} = \frac{1}{3} A_e$$

$$P_e = 0.4 \ P_{ox}$$

Let A^*_x & A^*_y represent the C.S.A where $M=1$ in upstream & downstream of the shock

For normal shock

$$T_{ox} = T_{oy}$$

$$\dot{m}_x = \dot{m}_y$$

$$\Rightarrow \frac{\dot{m}_x \sqrt{T_{ox}}}{A^*_x \ P_{ox}} = \frac{\dot{m}_y \sqrt{T_{oy}}}{A^*_y \ P_{oy}}$$

$$A^*_x P_{ox} = A_y^* P_{oy}$$

Here $A_x^* = A_{th}$

$$A_{th} P_{ox} = A_y^* P_{oy}$$

$$\frac{A_{th}}{A_y^*} = \frac{P_{oy}}{P_{ox}}.$$

In order to obtain relations as per values given in question following step is adopted

$$\frac{A_{th}}{A_y^*} \times \frac{A_e}{A_e} = \frac{P_{oy}}{P_{ox}} \times \frac{P_e}{P_e}.$$

as while
bream
ask

$$\frac{A_{th}}{A_e} \times \frac{A_e}{A_y^*} = \frac{P_{oy}}{P_e} \times \frac{P_e}{P_{ox}}$$

$$\frac{1}{3}\,\frac{A_e}{A_y^*} = \frac{P_{oy}}{P_e} \times 0.4$$

$$\frac{A_e}{A_y^*} \times \frac{P_e}{P_{oy}} = 0.4 \times 3 = 1.2$$

$$\left(\frac{A}{A^*} \times \frac{P}{P_o}\right)_{\text{out of nozzle}} = 1.2$$

from isentropic tables for $\gamma = 1.4$ $\frac{A}{A^*}\frac{P}{P_o} = 1.2$

$M_e = 0.472$

needs to be interpolated
0.47 (x_1) 1.205 (y_1)
x 1.2 (y)
0.48 (x_2) 1.198 (y_2)

$$\frac{P_e}{P_{oy}} = 0.859$$

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \Rightarrow x = 0.4718 = 0.472$$

$$P_{oy} = P_e / 0.859$$

$$\frac{P_{oy}}{P_{ox}} = \frac{P_e}{P_{ox}} \times \frac{1}{0.859} = \frac{0.4}{0.859} = 0.466$$

from normal shock table for $\gamma = 1.4$, $\frac{P_{oy}}{P_{ox}} = 0.466$

$\boxed{M_x = 2.58}$, $\boxed{M_y = 0.506}$

from isentropic tables for $\gamma = 1.4$, $M_x = 2.58$ — $\frac{A_x}{A_x^*} = 2.842$

$\frac{P_x}{P_{ox}} = 0.0517$

$$\frac{A}{A^*} = \frac{A}{A_{th}} = 2.842$$

$\therefore$ Area of cross section where shock occurs is $\boxed{(2.842 \times A_{th})}$

$$\frac{P_x}{P_{ox}} = 0.0517 \Rightarrow \boxed{P_x = 0.0517\, P_{ox}.}$$

gas while flowing through a nozzle encounters a shock. The Mach no. upstream of shock is 1.6 and static temperature downstream of shock is 470K. Calculate change in velocity across shock.

Sol: Given

$M_x = 1.6$

$T_y = 470 K$

$\gamma = 1.4$

$R = 287 \ J/kgK$ } not specified so....

For $\gamma = 1.4$, $M_x = 1.6$ from Normal shock tables

M_x	M_y	P_y/P_x	T_y/T_x	P_{oy}/P_{ox}	P_{oy}/P_x
1.6	0.668	2.82	1.388	0.895	3.805

downstream Mach number $M_y = 0.668$

$$\frac{T_y}{T_x} = 1.388 \Rightarrow T_y = 1.388 \times T_x.$$

$$T_x = (1.388)^{-1} \times 470 = 338.617 K.$$

$$M_x = \frac{C_x}{\sqrt{\gamma R T_x}} \Rightarrow C_x = M_x \times \sqrt{\gamma R T_x}.$$

$$C_x = 1.6 \times \sqrt{1.4 \times 287 \times 338.62}$$

$$C_x = 590.177 \ m/s$$

$$M_y = \frac{C_y}{\sqrt{\gamma R T_y}} \Rightarrow C_y = M_y \times \sqrt{\gamma R T_y}$$

$$C_y = 0.668 \times \sqrt{1.4 \times 287 \times 470}$$

$$= 290.288 \ m/s$$

$$C_x - C_y = 590.177 - 290.288 = 299.888 \approx \boxed{300 \ m/s.}$$